Autonomous Navigation for Flying Robots

Lecture 2.1: Recap on Linear Algebra

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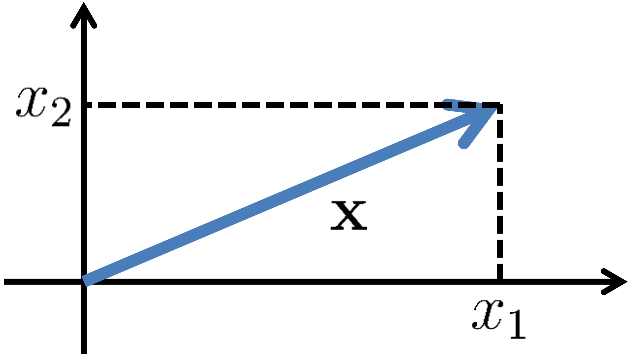
Notation

- Scalar $s \in \mathbb{R}$
- Vector $\mathbf{x} \in \mathbb{R}^n$
- Matrix $\mathbf{M} \in \mathbb{R}^{m \times n}$

Vectors

- Vector and its coordinates

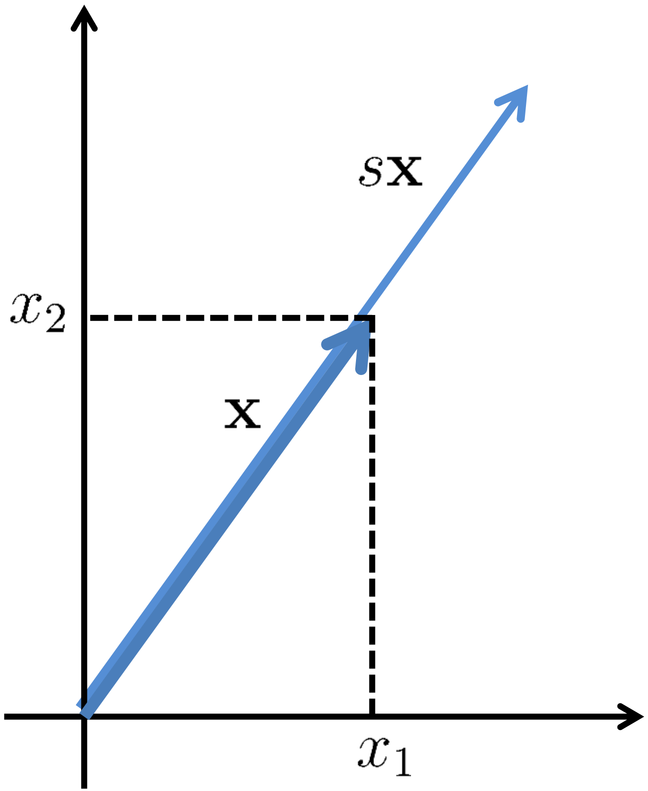
$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$$



- A vector represents a point in n-dimensional space

Operations on Vectors

- **Scalar multiplication**
- Addition/subtraction
- Length
- Normalized vector
- Dot product
- Cross product



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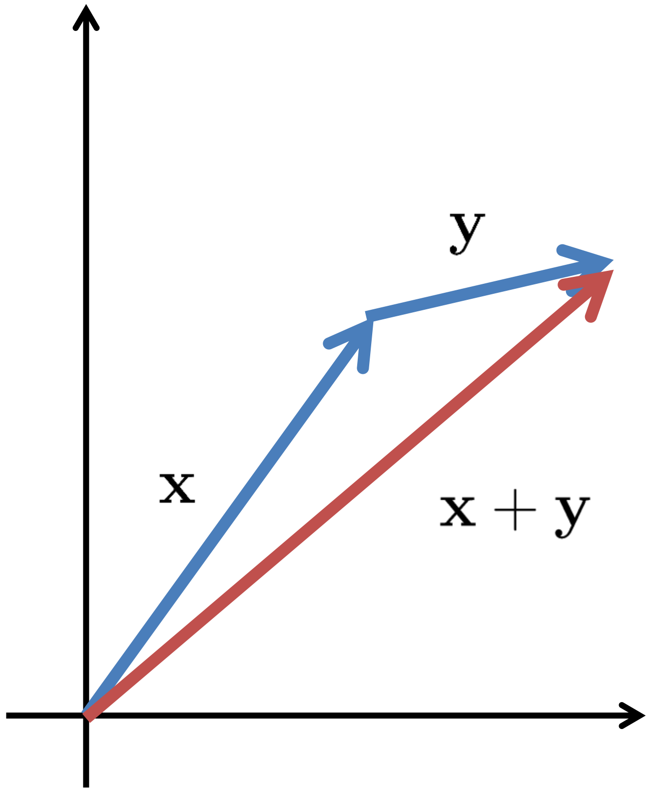

Operations on Vectors

- Scalar multiplication
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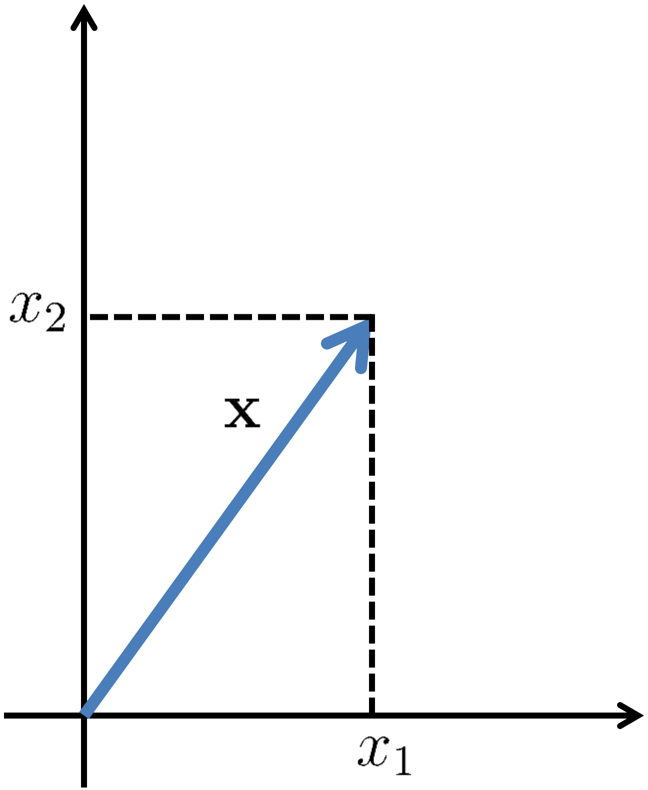
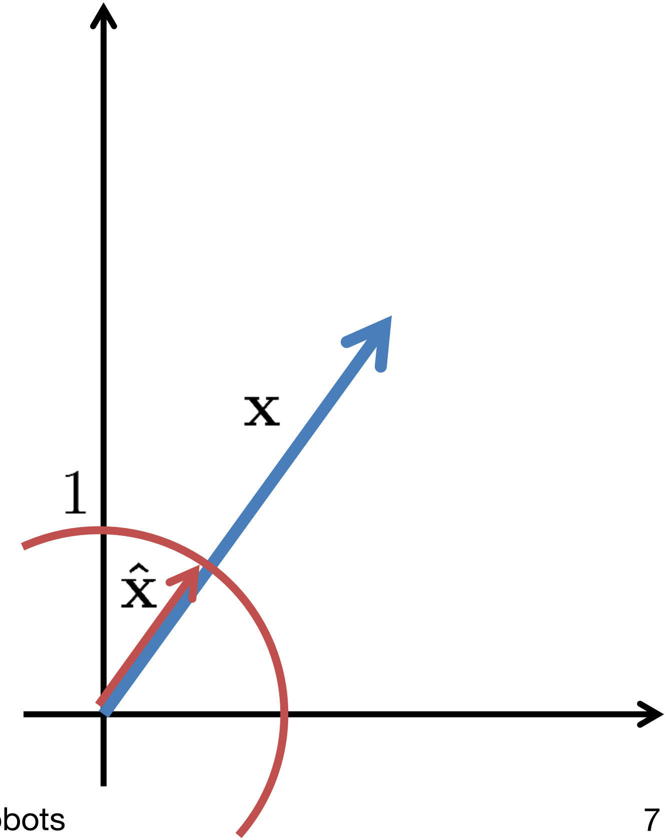
$$\|\mathbf{x}\|_2 = \|\mathbf{x}\| = \sqrt{x_1^2 + x_2^2 + \ldots}$$



Operations on Vectors

- Scalar multiplication
- Addition/subtraction
- Length
- **Normalized vector**
- Dot product
- Cross product

$$\hat{\mathbf{x}} = \frac{\mathbf{x}}{||\mathbf{x}||}$$

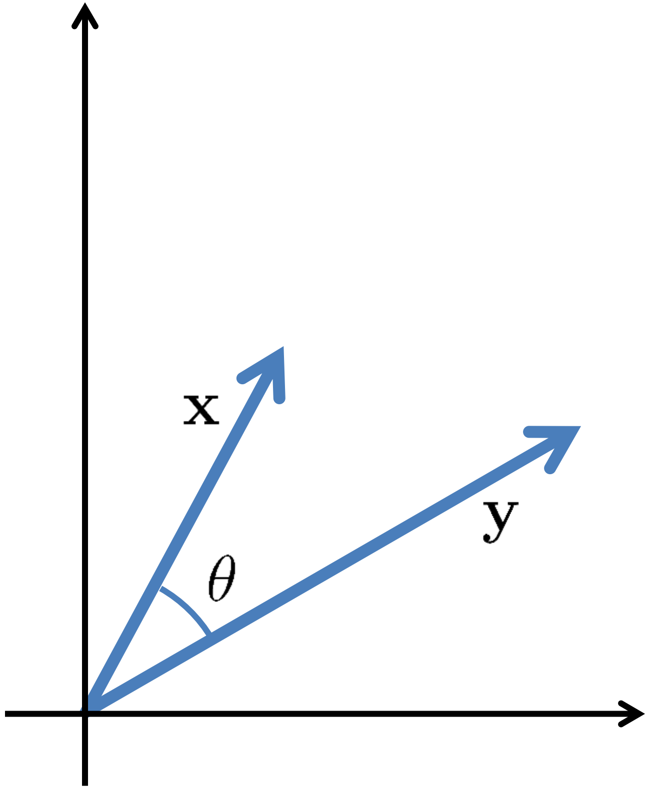


Operations on Vectors

- Scalar multiplication
- Addition/subtraction
- Length
- Normalized vector
- **Dot product**
- Cross product

$$\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\| \|\mathbf{y}\| \cos\theta$$

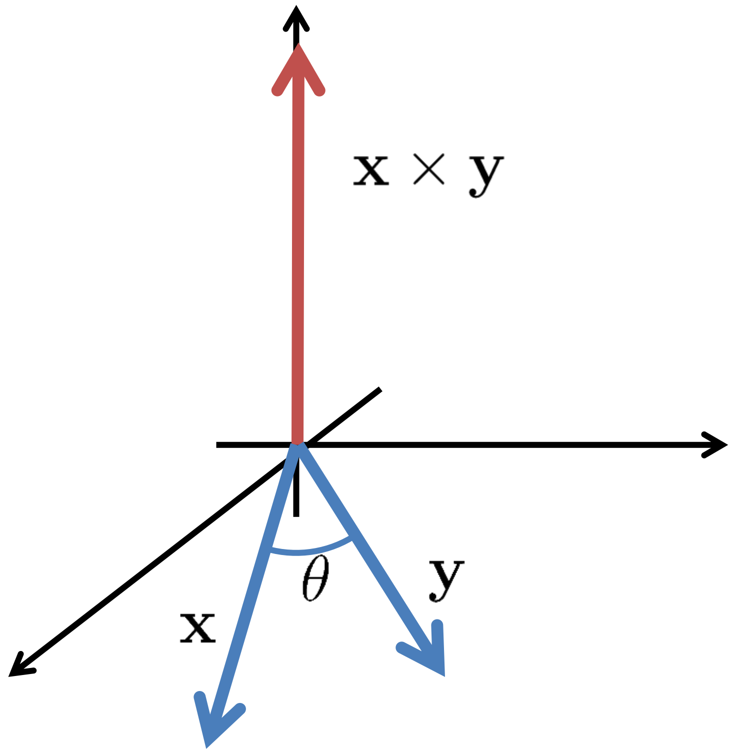
$\mathbf{x}, \mathbf{y}$ are orthogonal if $\mathbf{x} \cdot \mathbf{y} = 0$



Operations on Vectors

- Scalar multiplication
- Addition/subtraction
- Length
- Normalized vector
- Dot product
- **Cross product**

$$\mathbf{x} \times \mathbf{y} = \|\mathbf{x}\| \|\mathbf{y}\| \sin(\theta) \mathbf{n}$$



Cross Product

- Definition

$$\mathbf{x} \times \mathbf{y} = \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}$$

- Matrix notation

$$[\mathbf{x}]_\times = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

- Verify that $\mathbf{x} \times \mathbf{y} = [\mathbf{x}]_\times \mathbf{y}$

Matrices

- Rectangular array of numbers

rows columns

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & & & \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}$$

- First index refers to row
- Second index refers to column

Types of Matrices

- Square matrix
- Diagonal matrix
- Upper and lower diagonal matrix
- Symmetric matrix $\mathbf{X} = \mathbf{X}^\top$
- Skew-symmetric matrix $\mathbf{X} = -\mathbf{X}^\top$
- (Semi-)positive definite matrix $\mathbf{a}^\top \mathbf{X} \mathbf{a} \geq 0$
- Orthogonal matrix $\mathbf{X}^\top = \mathbf{X}^{-1}$

Operations on Matrices

- Matrix-vector multiplication $\mathbf{M}\mathbf{x}$
- Matrix-matrix multiplication $\mathbf{M}_1\mathbf{M}_2$
- Inverse $\mathbf{M}^{-1}$
- Transpose $\mathbf{M}^\top$
- Singular value decomposition $\mathbf{M} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\star$
- Eigendecomposition (eigenvalues and eigenvectors)

$$\mathbf{M} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1} \qquad \mathbf{A}\mathbf{v} = \lambda\mathbf{v}$$

Lessons Learned

- Notation used in this course
- Scalars, vectors, matrices
- Most important operations

- Next video: 2D and 3D geometry